



Solution of the multigroup neutron diffusion equations by the finite element method

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Title and author(s) Solution of the Multigroup Neutron Diffusion Equations by the Finite Element Method by Ib Misfeldt	Date 29th July 1975
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13 pages + tables + illustrations	Group's own registration number(s)
Abstract The finite element method is applied to the multigroup neutron diffusion equations. General formulas are given for Lagrange type elements. Description of some 2D- and one 3D computer code is given. Several investigations are described and the results are presented. The capability of the 3D code is demonstrated on the 3D IAEA Benchmark problem. Suggestions for further work are given.	Copies to
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1. SHORT ABOUT THE ELEMENT METHOD

1.1 General

I have worked on the solution of the multigroup diffusion theory equation by the finite element method.

The inhomogeneous equation for one energy group is

$$-\nabla(D(\underline{r})\nabla\phi(\underline{r})) + \Sigma_r(\underline{r})\phi(\underline{r}) - S(\underline{r}) = 0 \quad \text{in } \Omega \quad (1)$$

and

$$D(\underline{r}) \frac{\partial \phi(\underline{r})}{\partial n} + \gamma(\underline{r})\phi(\underline{r}) + q(\underline{r}) = 0 \quad (2)$$

on the boundary T of Ω .

In the finite element method the domain Ω is divided into a number of subdomains, Ω^e each corresponding to an element, e .

Within each element the flux, $\phi(\underline{r})$ is approximated by a polynomial, $\theta(\underline{r})$. The coefficients in the polynomial are connected with the flux or the derivative of the flux, at corner points and other fixed points (called nodes or flux-points) in the element. The flux in these fixed points (+derivative) is collected in a vector, $\underline{\delta}^e$ and the element interpolation function, $\underline{N}(\underline{r})$ is defined by the equation $\theta(\underline{r}) = \underline{N}^T(\underline{r}) \cdot \underline{\delta}^e$.

A characteristic for the element interpolation function is $N_i(\underline{r}_i) = 1$ and $N_i(\underline{r}_j) = 0$, $j \neq i$ where \underline{r}_i is the flux-point with flux (or derivative of the flux) corresponding to δ_i^e .

It is shown (ref. 1) that the one group diffusion equation (1) is the Euler equation for the functional

$$\begin{aligned} J[\phi] = & 1/2 \int_{\Omega} (D [\nabla \phi]^T [\nabla \phi] + \Sigma_r \phi^2 - 2\phi S) d\Omega \\ & + 1/2 \int_T (\gamma \phi^2 + 2\phi q) d\tau. \end{aligned} \quad (3)$$

The term $S(r)$ gives the coupling between the groups.

In variational calculation it is shown that the best approximation to the solution of the equations (1) + (2) within a given class of functions is the function which minimizes the functional (3). With the finite element approximation this minimum requirement leads to the equations

$$\underline{H} \underline{A} + \underline{F} = \underline{0} \quad (4)$$

where

$$(\underline{H})_{rs} = \sum_e (H^e)_{rs} \quad r, s = 1, 2, \dots, M$$

$$(\underline{F})_r = \sum_e (F^e)_r \quad r = 1, 2, \dots, M$$

the summations run over all elements in Ω , M is the total number of nodes (flux-points),

$$\underline{H}^e = D^e \int_{\Omega^e} (\underline{\nabla} \underline{N}^T)^T (\underline{\nabla} \underline{N}^T) d\Omega + \underline{L}_r^e \int_{\Omega^e} (\underline{N} \underline{N}^T) d\Omega + \gamma^e \int_{T^e} (\underline{N} \underline{N}^T) d\tau,$$

$$\underline{F}^e = - \int_{\Omega} \underline{N} S d\Omega + q^e \int_{T^e} \underline{N} d\tau$$

$$= \sum_{g'=1}^G (S_{g' \rightarrow g} \cdot \int_{\Omega^e} \underline{N} (\underline{N}^T \underline{S}_{g'}) d\Omega) + q^e \int_{T^e} \underline{N} d\tau$$

G = total number of groups

$$S_{g' \rightarrow g} = (\chi_{S, g' \rightarrow g} + \frac{1}{K_{eff}} \chi_g (\chi \Sigma_f)_{g'}),$$

T^e is the external boundary for the element e . This is shown in details in ref. 2.

To solve the equations (4) one must know the element interpolation function, $\underline{N}(r)$. When the interpolation function is fixed the integrals can be evaluated and the system of linear equations

(4) can be solved.

1.2 Element interpolation functions

I have decided only to consider element interpolation function of Lagrange type. They use only the flux and not the derivatives as parameters, which for me seems more reasonable than involving derivatives leading to difficulties at cornerpoints between materials with different diffusion coefficients, though the difficulties have been treated (e.g. in ref. 6), it is only possible for rectangular mesh and I did not want to exclude the improvement obtainable by triangular mesh.

Fig. 1 shows Lagrange triangles and rectangles of dimension $m = 1, 2$ and 3 .

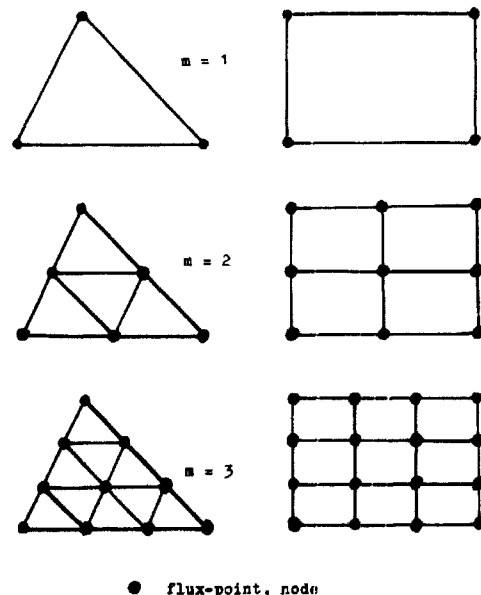


Fig. 1. Lagrange triangles and rectangles of order 1, 2, and 3.

If we consider a triangle in the barycentric coordinate system, defined on fig. 2, we can easily show that $N(L_1, L_2, L_3)$ for a Lagrange triangle of order m is defined by

$$N_i = N_{p,q,r}(L_1, L_2, L_3) = \frac{1}{p!q!r!} \sum_{s=0}^p \sum_{t=0}^q \sum_{u=0}^r \frac{L_1^s \cdot L_2^t \cdot L_3^u}{s!t!u!} \cdot S_p^{(s)} \cdot S_q^{(t)} \cdot S_r^{(u)}$$

$$S_p^{(s)} = \frac{p!}{s!(p-s)!} \cdot S_p^{(s-1)}$$

$S^{(j)}$ is the Stirling number of the 1st kind,

$P_i = (\frac{L_1}{m}, \frac{L_2}{m}, \frac{L_3}{m})$ is the fixed fluxpoint corresponding to N_i , the point with the flux δ_i . This is shown in details in ref. 2.

For a rectangle of order m the used coordinate system is natural coordinates as defined on fig. 3.

The interpolation function $N(\xi, \eta)$ is defined by

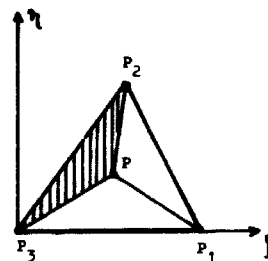
$$N_i = N_{p,q}(\xi, \eta) = f_p(\xi) \cdot f_q(\eta)$$

$$f_p(x) = \frac{(-1)^{m-p}}{p!(m-p)!} \sum_{j=0}^m T_{m,p}^{(j)} (mx)^j$$

where

$$T_{m,p}^{(j)} = S_{m+1}^{(j+1)} + p \cdot T_{m,p}^{(j)}; \quad T_{m,p}^{(m)} = 1$$

with these interpolation functions the equation (4) is easily set up.

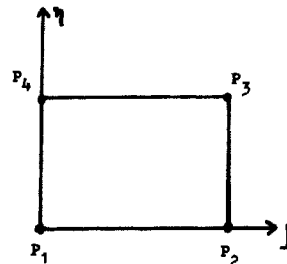


$$P = (L_1, L_2, L_3)$$

$$L_1 = \frac{\text{area } P P_2 P_3}{\text{area } P_1 P_2 P_3} = \frac{\text{hatched area}}{\text{total area}}$$

analogous for L_2 and L_3

Fig. 2. Barycentric coordinates.



$P_3 = (1, 1)$ in natural coordinates
analogous for P_1, P_2 and P_4

Fig. 3. Natural coordinates

2. THE DEVELOPED PROGRAMS

During my work with the solution of the multigroup diffusion theory equation I have developed several programs. In the following chapter I will give a short description of the most important versions.

2.1 FEMA

The program uses Lagrange triangles of arbitrary order, the group equations are solved by direct methods taking advantages of the sparsity in the matrices involved.

This method seemed very effective until the number of unknowns exceeded 1000 then the execution times rose drastically, in fact I could not exceed 1400 unknowns in a night's calculation.

The program is described in details in ref. 2.

2.2 Problems to be solved before the extension to 3D.

The number of unknowns for a realistic 3D calculation was expected to be 10 000 - 30 000 or even more, therefore it was clear that new solution techniques had to be found.

An other problem was the element shape, if I continued with the triangular shape (tetrahedron) in three dimensions the input-preparation would become very complicated unless the program could automatically divide box-formed meshes into tetrahedrons.

A third problem was the very large matrices that arise from the finite-element approximation in 3D calculations, as an example the matrix (H) for a 3rd order calculation with about 10 000 unknowns per group contains about 10^6 elements $\frac{1}{3}$ 0 per group.

2.3 FEMB

The solution of the problems was carried through in 2 dimensions, leading to the program FEMB.

The direct solution of the group equations was replaced by a pointwise successive overrelaxation. This method was found to be superior to the direct solution in most cases, only in very small unrealistic examples, the direct solution was faster. I can refer to a calculation which I and G.K. Kristiansen performed on a German

Benchmark problem, representing a reactor where one control rod was accidentally removed, leading to a very high local reactivity. On this example the second eigenvalue was very near 1 which made it practically impossible for the direct method to converge (more than 1000 outer iterations were needed even though extrapolation was used). With a good overrelaxation factor the iterative solution converged on less than 100 outer iterations with one inner per outer iteration. If I performed many (5 - 20) inner iterations per outer the same difficulty as with the direct solution arose.

I decided to use boxformed elements in the 3D program instead of tetrahedrons, this leads to a loss in flexibility, but the program became much easier to use.

The rectangular elements were examined in two dimensions and I found about the same accuracy as if the rectangles were divided into two triangles.

The third problem, the large matrices, was handled in the following manner. I did naturally take full advantage of the sparsity of the matrices, furthermore their elements are stored on backing storage and accessed absolutely sequentially during the iterations, but during the set up of the matrices I access the element unordered. This did run very well even for very large 2D problems and on the basis of this 2D program I decided to develop the 3D version.

During the 3D calculations I soon ran into problems because of the large matrices, during the set up of the matrices. I did solve these problems in a rather untraditional way, but on computers with more backing storage these problems could easily be solved in a more efficient way.

3. INVESTIGATIONS CARRIED THROUGH

The goal for my investigations with finite element methods was, if possible, to develop a fast and accurate 3D-fluxcalculation program.

There had already been done a lot of work in this field with 2D-codes, for example the investigations described in ref. 3, ref. 5 and ref. 6. What I missed in these investigations was realistic

examples on light water reactors, which mainly is the field in which we are working at Rise. An other lack was flux comparisons, I do not believe that a high precision on the eigenvalue necessarily means a high precision on the calculated flux.

3.1 First investigation

The first investigation, carried through with FEM, is described in details in ref. 2. The goal was to complete the investigations found in the literature with respect to our interests. On basis on this we should decide whether or not we would work on a 3D program.

I only examined the accuracy of the solution as function of meshsize for a rectangular division because of the expected difficulties with tetrahedrons in a 3D calculation.

The investigation included 2 realistic examples, a 2 group calculation on the midplane of the Connecticut Yankee reactor and a 2D version of the 3D IAEA Benchmark. Further I included 3 smaller examples with varying discontinuities caused by different diffusion coefficients in core and reflector.

The result of this investigation was.

- 1) On "nice" problems (small difference in diffusion coefficients) the high order finite element approximations provide a more accurate solution within the same execution time, but with increasing discontinuities the accuracy of the high order methods decrease faster than for the low order methods (FDT as well as FEM 1st order).
- 2) The realistic problems seemed to be "nice" enough for the high order methods to converge faster (as function of meshsize) than the low order methods when considering calculation times, but the improvements were less than expected at the beginning of the investigation.

All in all I did from my first investigation conclude that the finite element method does not constitute an alternative to the fast nodal methods, but the method is superior to the low order FDT programs with respect to calculation time when high precision is wanted.

3.2 Second investigation

The next phase in my work was the development of the program FEMB as described in chapter 2.

With this program I performed some calculations on planes in the 3D IAEA Benchmark problem in order to estimate the precision of a calculation on the 3D IAEA Benchmark problem.

One of the considered planes was the core midplane, where I naturally used the 2D IAEA specification as published in ref. 7. As reference solution I used an extrapolated FDT solution performed by G.K. Kristiansen. The results from this calculation are described in appendix A.

This investigation showed that my first investigation was too optimistic in its conclusions considering FEM, but still the FEM program seemed to be a good deal faster than the FDT programs, so I continued the work with the 3D version based on the program FEMB.

3.3 Third investigation

After the development and testing of the 3D program I had planned some calculations on the 3D IAEA Benchmark problem. The goal of these calculations was to provide a reliable reference solution to the problem. In order to get a good estimate of the errors in the flux-distribution I had performed the described 2D calculations.

The largest calculation on the 3D IAEA Benchmark problem is described in appendix B, it is a calculation with $16 \times 16 \times 13$ mesh and a second order approximation. The mesh used in the directions is chosen so that the max. error in the planes considered (in 2D calculations), was about 3.5 % (relative to ϕ_{\max} in the group). according to the 2D calculations I assume the max. error to be less than 5% on the 3D calculation described in appendix B.

This calculation is until now the most accurate calculation performed on this problem. Naturally there will be some doubt about this result, because the program is new, including risk for errors, and the method is not too well known considering accuracy for realistic examples, especially not in three dimensions. I hope that a large FDT calculation with comparable accuracy will soon be available and support my results and estimations.

I performed two further calculations on the problem, a 2nd

order and a 3rd order coarse mesh calculation, the result from the 3rd order calculation is shown in appendix C.

3.4 Comparative study on the way

A comparative study of different method, FDT-cornerpoint, FDT-midpoint and FEM is at the moment carried out by Mr. G.K. Kristiansen, Rise. The results of this work will soon be available.

4. FUTURE WORK

In the future work with FEM programs we will concentrate on programs suitable for 3D-overall calculations.

Until now we have in no way taken advantage of improvements, which can be achieved by the flexible division of the reactor with a triangular mesh. The gain obtained by this seems to be quite large (see f.ex. ref. 3 and ref. 5).

We have planned to develop a 3D-code using prism-formed elements (triangles in the xy-plane, rectangles in the xz- and the yz-plane). This should at the expense of some manual work be able to reduce the calculation time.

The last part of the work with the 3D-codes is planned to be an optimisation of the code.

There are several ways which can lead to a faster code, I will mention some.

Most of the calculation time is spent during the iterations, here the solving of the equations and the calculation of the right side (\bar{F}) takes almost all the time. The calculation of \bar{F} can be accelerated sacrificing some generality (scattering from all groups to all groups, fission neutrons born in all groups). If the order of approximation is fixed, a lot of calculation can be saved, I have tried this for 1st order approximation, leading to a saving of about 80% cpu-time. The gain will be less in higher order calculations, I expect about 50% for 2nd order calculation etc.

An other way to accelerate the calculation of \bar{F} is at the expense of more storage, here the gain should be about 50%, but at the moment this would not be advisable at our computer.

The total number of iterations might be reduced by introduction of more efficient iteration techniques. But as long as the simple pointwise successive overrelaxation is able to finish within about 100 iterations, it will probably be difficult to find a much faster method.

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APPENDIX A

Danish Atomic Energy Commission
Research Establishment Risø
Department of Reactor Technology
Reactor Physics Section

RP-3-75
4 June 1975
IM/rj
NEACRP-L-138

2D IAEA Benchmark problem

by

Ib Misfeldt

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In connection with the calculation on the 3D Benchmark problem we have performed some 2D-calculations. The 2D problem is the one specified in ref. 1 from which fig. 1 is copied.

To get a reasonably accurate independent reference solution Mr. G.K. Kristiansen has performed a series of FDT-calculations and from them obtained a solution by Richardson-extrapolation in each power assembly.

Table 1 shows the results from these calculations. In table 1 are further shown some calculations with **FEMB**, a two-dimensional finite element flux calculation program using a rectangular mesh. From ref. 1 are taken two calculations with **FEM2D**, a finite element flux calculation program using a triangular mesh. The errors shown in Table 1 are defined:

$$\xi_r = \max_i \left(\frac{|P_i - P_{i,ref}|}{P_{i,ref}} \right) \times 100$$

$$\xi_{abs} = \max_i \left(\frac{|P_i - P_{i,ref}|}{P_{max,ref}} \right) \times 100$$

(P_i is the assembly power, and i denotes the assembly)

The error in the flux, defined as:

$$\xi = \max_r \left(\frac{\max_i |\phi_i^r - \phi_{i,ref}^r|}{\phi_{max,ref}^r} \right) \times 100$$

(i denotes the spacepoint, r is a group-index)

is typically found to be twice as big as ξ_{abs} in the power.

In fig. 2 the error as a function of the mesh is shown for the different calculations.

Solution No		description	B 6700 cpu-time
1	tvedin	17 x 17	1.4 min.
2	"	34 x 34	7.1 "
3	"	68 x 68	30 "
4	"	136 x 136	110 "
5	"	170 x 170	145 "
6		extrapolated value	
7	FEMB	2. order 9 x 9 (19 x 19) ^a	4.4 "
8	"	2. order 18 x 18 (37 x 37) ^a	23.3 "
9	"	2. order 36 x 36 (73 x 73) ^a	98.5 "
10	FEM2D	2. order 182 fluxpoints 1/8 core	
11	"	2. order 606 fluxpoints 1/8 core	
12	FEMB	3. order 9 x 9 (28 x 28) ^a	18.9 "
13	"	3. order 18 x 18 (55 x 55) ^a	90.6 "

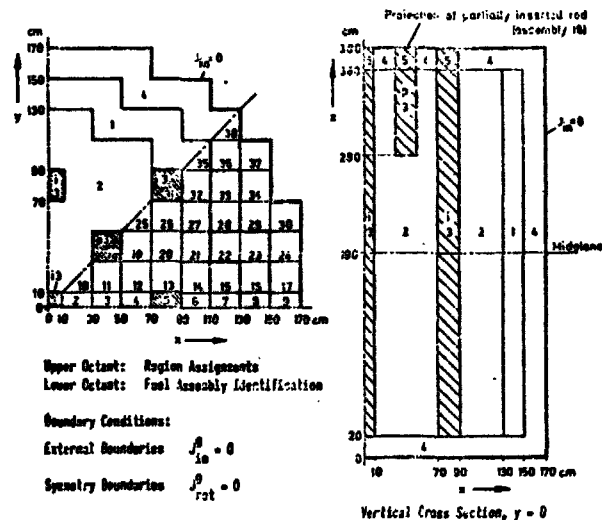
TVEDIN is a FDT fluxcalculation program using corner mesh points.
FEMB is a finite-element flux calculation program using Lagrange interpolation in a rectangular mesh.

FEM 2D is a finite element flux calculation program using Lagrange interpolation in a triangular mesh. See ref. 1.

^a 9 x 9 (19 x 19) means 9 mesh in each direction and 19 fluxpoints because of the second order approximation etc.

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April 15 - 17, 1975
SESSION I
M.R. WAGNER: Current Trends in Multidimensional Static Reactor Calculations.



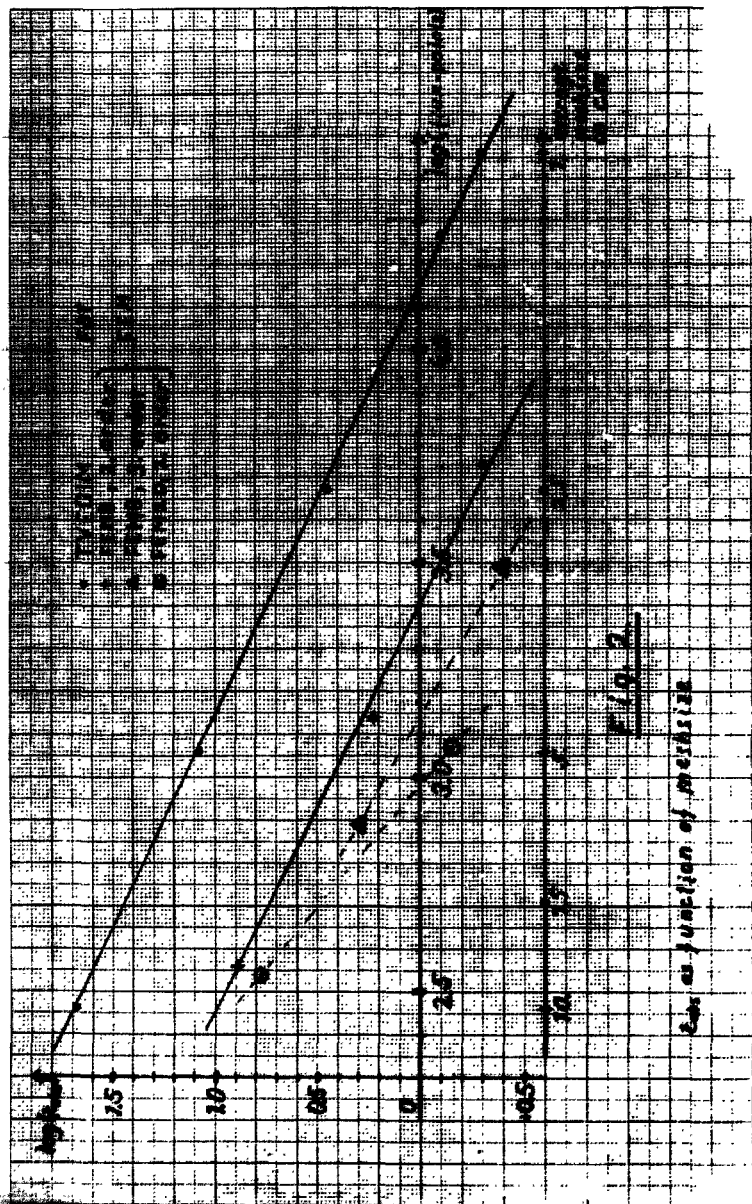
Group Constants for 3D IAEA Benchmark Problem

Region	ρ_1	ρ_2	Σ_{1+2}	Σ_{a1}	Σ_{a2}	$\nu\Sigma_{f2}$	
1	1.5	0.4	0.02	0.01	0.08	0.135	Fuel 1
2	1.5	0.4	0.02	0.01	0.085	0.135	Fuel 2
3	1.5	0.4	0.02	0.01	0.13	0.135	Fuel 2 + Rod
4	2.0	0.3	0.04	0	0.01	0	Reflector
5	2.0	0.3	0.04	0	0.055	0	Ref. + Rod

$$X_1 = 1.0, X_2 = 0.0, \nu\Sigma_{f1} = 0 \text{ all regions}$$

Note: 3D IAEA Benchmark Problem represents midplane: $x = 170$ cm with constant axial buckling $k^2 = 0.8 \times 10^{-6}$ for all regions and energy groups

Fig. 1 3D IAEA Benchmark Problem Specification



method program points sol.no.	FEM FEM2D					FEM FEM3D				FEM FEM3D			
	1x17	3x34	1x17	3x34	1x17	1x17	3x34	1x17	3x34	1x17	3x34	1x17	3x34
1	478	654	783	1000	1200	1400	1600	1800	2000	2200	2400	2600	2800
2	729	1147	1267	1608	1800	2000	2200	2400	2600	2800	3000	3200	3400
3	870	1301	1415	1600	1800	2000	2200	2400	2600	2800	3000	3200	3400
4	760	1088	1178	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200
5	532	601	610	610	610	610	610	610	610	610	610	610	610
6	867	910	930	940	940	940	940	940	940	940	940	940	940
7	1083	978	946	937	946	946	946	946	946	946	946	946	946
8	1279	879	786	753	760	760	760	760	760	760	760	760	760
10	830	775	734	710	710	710	710	710	710	710	710	710	710
11	908	1333	1442	1470	1470	1470	1470	1470	1470	1470	1470	1470	1470
12	869	1207	1236	1307	1311	1311	1311	1311	1311	1311	1311	1311	1311
13	810	1002	1051	1065	1067	1067	1067	1067	1067	1067	1067	1067	1067
14	986	1030	1035	1036	1036	1036	1036	1036	1036	1036	1036	1036	1036
15	1136	1003	964	940	943	943	943	943	943	943	943	943	943
16	1267	862	768	744	744	744	744	744	744	744	744	744	744
18	952	1339	1437	1461	1464	1464	1464	1464	1464	1464	1464	1464	1464
19	947	1248	1321	1333	1342	1342	1342	1342	1342	1342	1342	1342	1342
20	959	1129	1167	1176	1178	1178	1178	1178	1178	1178	1178	1178	1178
21	1077	1081	1073	1071	1071	1071	1071	1071	1071	1071	1071	1071	1071
22	1248	1047	993	980	987	987	987	987	987	987	987	987	987
23	1395	861	735	703	700	700	700	700	700	700	700	700	700
25	895	1121	1175	1188	1190	1190	1190	1190	1190	1190	1190	1190	1190
26	819	930	757	964	966	966	966	966	966	966	966	966	966
27	988	934	914	908	908	908	908	908	908	908	908	908	908
28	1353	977	880	855	852	852	852	852	852	852	852	852	852
31	547	498	479	473	472	472	472	472	472	472	472	472	472
32	867	728	695	688	687	687	687	687	687	687	687	687	687
33	1209	743	634	607	603	603	603	603	603	603	603	603	603
35	1132	721	620	594	591	585	585	585	589	589	589	589	589
P ₀	102	24	6	1.6	1.0	-	17	3.6	1.0	15	1.7	4.2	0.9
P ₀₀	48	12	2.8	0.8	0.5	-	8	1.7	0.5	6	0.7	2.0	0.4

Table 1

2D IAEA BENCHMARK PROBLEM

Comparison of finite element- and
finite difference-methods.

Average assembly powers.

APPENDIX B

Danish Atomic Energy Commission
Research Establishment Risø
Department of Reactor Technology
Reactor Physics Section

RP-4-75
4 June 1975
IM/rj
NEACRP-L-142

3D IAEA Benchmarkproblem
2nd order FEM calculation

by

Ib Misfeldt

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Code name FEM 3D
Organisation Risø. DK.

3D Benchmark Problem

Result Scheme 1, Page 1

(to be filled in by type-writer)

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2. Computer code name: FEM 3D

Description (max 20 lines):

FEM 3D is a three-dimensional finite-element flux calculation programme. The programme uses box-formed elements with Lagrange interpolation. The order of the interpolation might be 1., 2. or 3. order. The programme uses an ordinary power iteration technique with one inner iteration per outer. The group equations are solved by SOR and the iterations are accelerated by extrapolation.

References:

None so far.

3D Benchmark Problem

Result Scheme 1, Page 2

(to be filled in by type-writer)

3. Calculation description (simplifications, meshes, iterations, problems, etc.) (max. 15 lines):

This solution is performed with 2. order interpolation. The mesh size is $16 \times 16 \times 13$ ($33 \times 33 \times 27$ flux points) and 2 energy groups are used.

The result scheme is filled in after 71 iterations and the local error is assumed to be less than 0.1% of ϕ_{\max} in each group.

4. Computer type: Burroughs 6700

Calculation time: 23 hours cpu

Computer speed relative to one or two well-known computer types:

1 hour on CDC 6600 \sim 10 hours on B 6700.

An extra page A4 with information may be added if necessary.

The precision of this calculation

It is possible to estimate the precision of this 3D-calculation from some 2D-calculations.

These 2D-calculations are described in RP-3-75 (ref. 1). It is seen that the accuracy of a calculation with the mesh used in the 3D-calculation is about 1.7% on the assembly power in the xy-plane, this corresponds to about 3.5% in the flux in each group.

I have also performed 2D-calculations in the plane $y = 40$ and found that the mesh used in the 3D-calculation in the z-direction gives about the same accuracy as described above.

From these considerations I will estimate the absolute error to be less than 5% in the whole reactor.

A FDT-calculation with the same accuracy in the flux is in ref. 1 seen to need about 100×100 mesh in the xy-plane. I will assume that the same accuracy can be obtained with about 100 mesh in the z-direction with a nonuniform mesh, where the meshsize around the core-reflector boundaries is not greater than in the xy-plane.

A FDT-calculation with comparable accuracy will therefore need about 10^6 mesh.

Also from ref. 1 it is seen that considerable improvements can be obtained with the use of triangular elements in the xy-plane, the same accuracy is obtained with almost doubled meshsize.

Ref. 1: RP-3-75.

3D Benchmark Problem

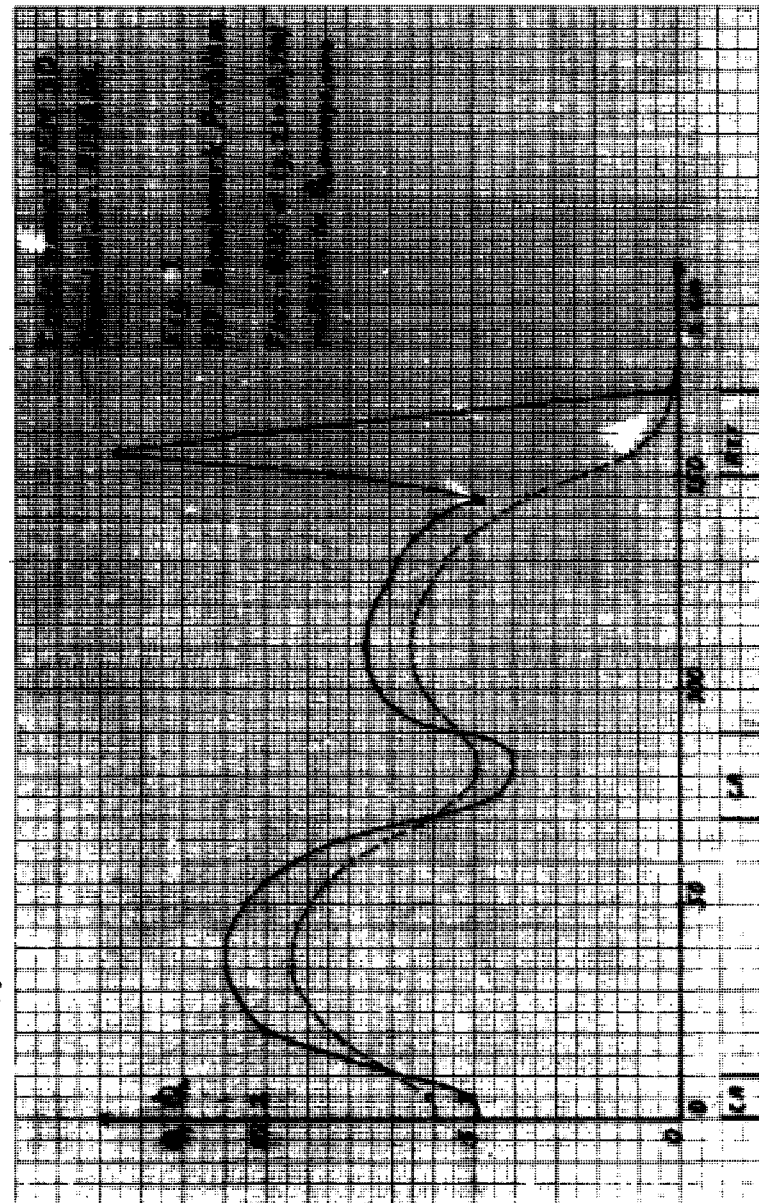
Results Scheme 2, k_{eff} and flux points

$$k_{eff} = 1.0292$$

Write ϕ_1 and ϕ_2 relative to $\phi_{r, average, core}$

	Co-ordinates cm			Relative fluxes	
	x	y	z	ϕ_1	ϕ_2
core midplane	80	40	190	7.743	1.816
	80	80	190	2.846	0.641
	100	80	190	4.499	1.108
	120	40	190	6.326	1.568
10 cm's up the partially inserted control absorber	0	0	290	2.656	0.444
	40	0	290	4.151	0.973
	80	40	290	3.815	0.895
	140	0	290	2.504	0.649
	150	0	290	1.271	0.941
Core bottom	0	0	20	0.613	0.370
	0	40	20	0.925	0.659
Power peak	130	56	178	3.291	2.426
	32	32	174	9.805	2.298

Relative fluxes and position, where ϕ_2 has its maximum in the core

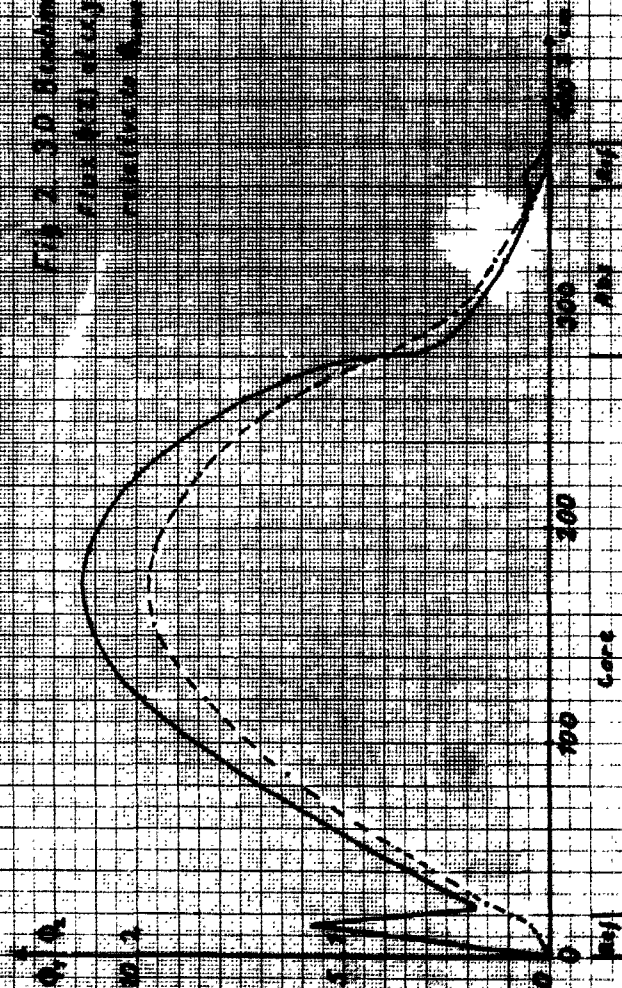


APPENDIX C

Danish Atomic Energy Commission
 Research Establishment Risø
 Department of Reactor Technology
 Reactor Physics Section

RP-5-75
 6 June 1975
 IM/rj
 NEA/RP-1-139

Fig. 2. 3D Benchmark Problem
 (100, 100, 100) (100, 100, 100)
 relative dimensions



3D IAEA Benchmarkproblem
 3rd order FEM calculation

by

Ib Misfeldt

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Code name FEM 3D
Organisation Riss. DK.

3D Benchmark Problem

Result Scheme 1, Page 1

(to be filled in by type-writer)

1. Name of participant: Ib Møsfeldt

Organisation: Danish Atomic Energy Commission

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2. Computer code name: FEM 3D

Description (max 20 lines):

FEM 3D is a three-dimensional finite-element flux calculation programme. The programme uses box-formed elements with Lagrange interpolation. The order of the interpolation might be 1., 2. or 3. order. The programme uses an ordinary power iteration technique with one inner iteration per outer. The group equations are solved bySOR and the iterations are accelerated by extrapolation.

References:

None so far.

Code name FEM 3D
Organisation Riss DK

3D Benchmark Problem

Result Scheme 1, Page 2

(to be filled in by type-writer)

3. Calculation description (simplifications, meshes, iterations, problems, etc.) (max. 15 lines):

This calculation is performed with 3rd order interpolation. The mesh used was $9 \times 9 \times 4$ ($28 \times 28 \times 13$ flux points). The solution is not yet sufficiently converged so the local error is large, but less than 5%.

The mesh in the z direction is too coarse so the precision will not be as high as predicted in RP-3-75 (6%) for the xy-plane.

These results do mostly serve the purpose to prove that 3rd order FEM calculation is possible on a realistic 3D-problem.

4. Computer type: Burroughs 6700

Calculation time: 9.5 hours

Computer speed relative to one or two well-known computer types:

1 hour on CDC 6600 ~ 10 hours on B 6700.

An extra page A4 with information may be added if necessary.

Code name FEM 3D
Organisation Rise DK

3D Benchmark Problem

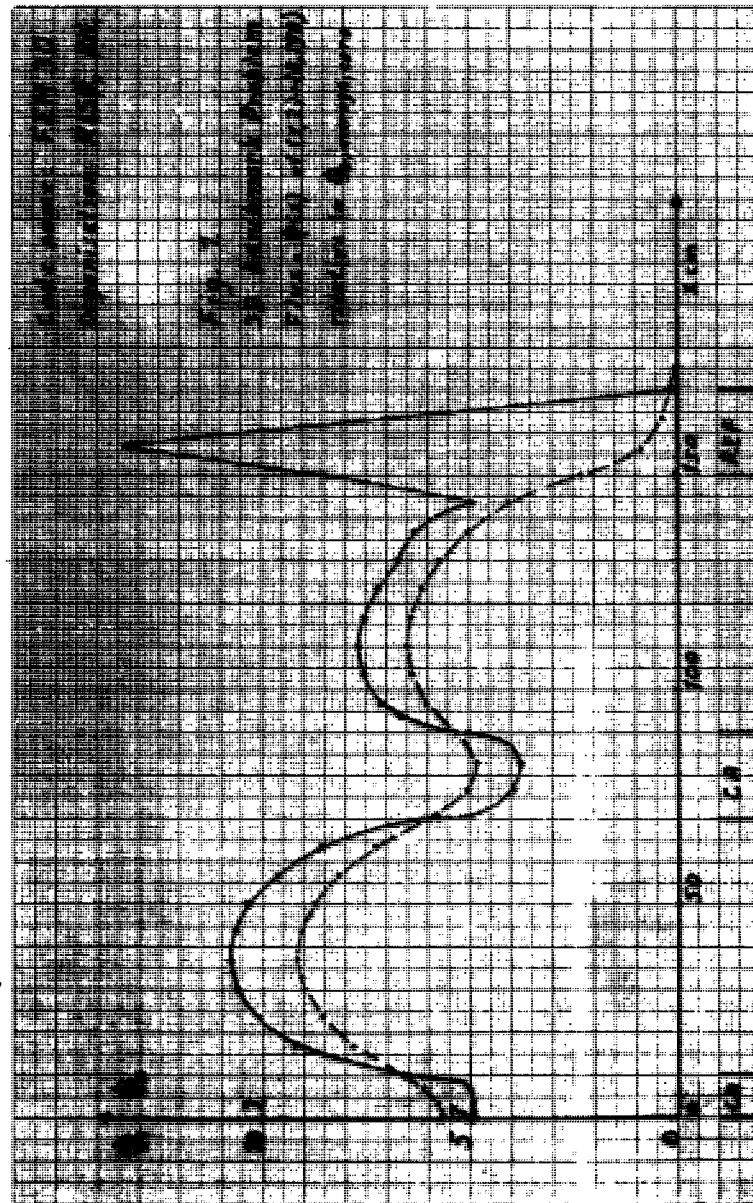
Results Scheme 2, k_{eff} and flux points

$$k_{\text{eff}} = 1.0287$$

Write ϕ_1 and ϕ_2 relative to $\phi_{2, \text{average, core}}$

	Co-ordinates cm			Relative fluxes	
	x	y	z	ϕ_1	ϕ_2
core midplane	80	40	193	7.626	1.796
	80	80	196	3.899	0.602
	100	80	190	4.395	1.095
	120	40	198	6.289	1.541
10 cm's up the partially inserted control absorber	0	0	293	2.760	0.426
	40	0	290	4.246	0.988
	80	40	290	3.924	0.914
	140	0	299	2.624	0.646
	150	0	290	1.339	0.984
Core bottom	0	0	20	692	0.150
	0	40	20	1.092	0.346
Power peak	130	56	175	3.229	<u>2.389</u>
	34	34	175	9.481	2.250

Relative fluxes and position, where ϕ_2 has its maximum in the core





Nr. 247

Code name: FEM 3D

Organisation: Riss, DK.

Fig. 2. 3D Benchmark problem

Flux $\Phi(Z)$ at $(X,Y) = (40, 40)$
relative to Φ_0 , average, core

